is "Lect. opt. London 1729. P. I., Sect. II, Art. 31." For oblique deviation, the fact that $v_4 = v_1$ was published by A. Bravais in 1845, *Journal de l'Ecole Polytechnique*, vol. 18, 30$^e$ Cahier, p. 79. The priority for formula (5) seems to belong to Mascart. This equation is given correctly in his *Traité d'Optique*, vol. 1, p. 84 (1889). By drawing an incorrect diagram, R. S. Heath (*l.c.* p. 32) derived the formula

$$\cos \tfrac{1}{2}D = \cos \tfrac{1}{2}D' \cos v_1.$$

This was copied by almost all later writers regardless of the fact that J. Larmor called attention to the error in the *Proceedings of the Cambridge Philosophical Society*, vol. 9, p. 108 (1896). An unbiased discussion of this matter (including the part which the present writer has taken) is given in Southall's treatise, p. 127. Finally, Konen in attempting to generalize formula (5) extended the error even farther by stating that the cosine equation still held (*l.c.* p. 267).

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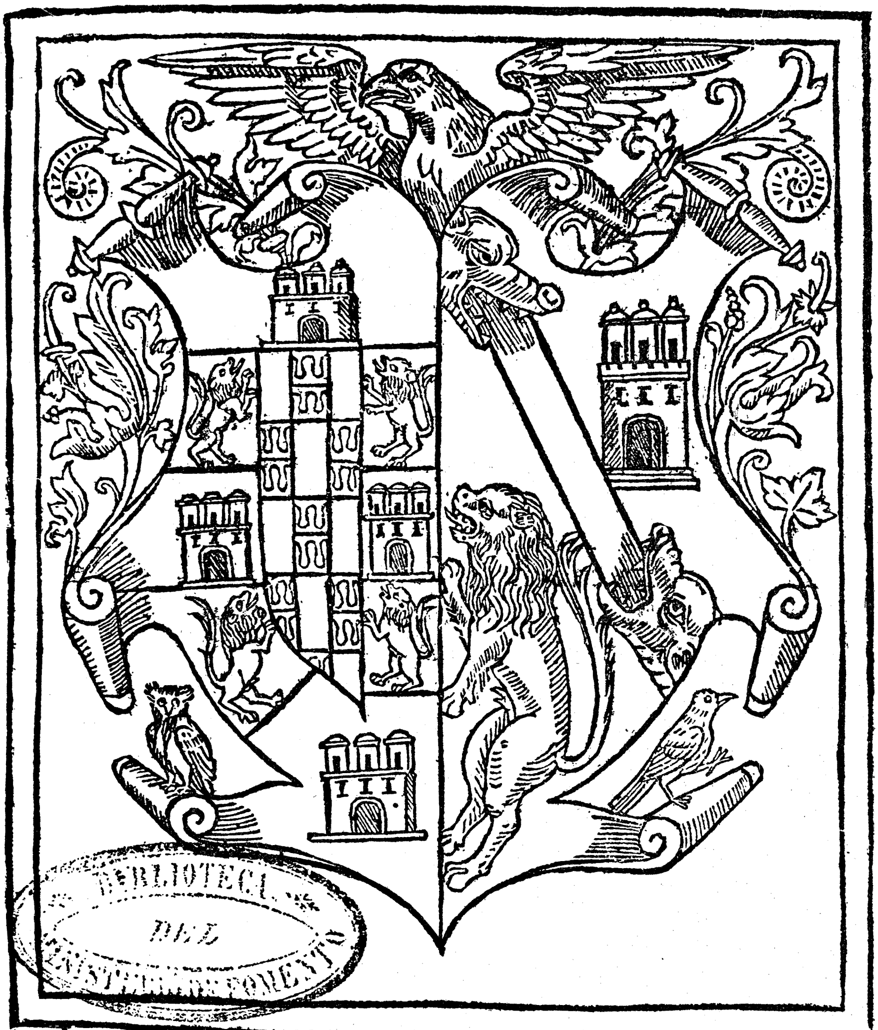
# THE FIRST WORK ON MATHEMATICS PRINTED IN THE NEW WORLD.[1]

By DAVID EUGENE SMITH, Columbia University.

**I. General Description.** If the student of the history of education were asked to name the earliest work on mathematics published by an American press, he might, after a little investigation, mention the anonymous arithmetic that was printed in Boston in the year 1729. It is now known that this was the work of Isaac Greenwood who held for some years the chair of mathematics in what was then Harvard College. If he should search the records still further back, he might come upon the American reprint of Hodder's well-known English arithmetic, the first textbook on the subject, so far as known, to appear in our language on this side of the Atlantic. If he should look to the early Puritans in New England for books of a mathematical nature, or to the Dutch settlers in New Amsterdam, he would look in vain; for, so far as known, all the colonists in what is now the United States were content to depend upon European textbooks to supply the needs of the relatively few schools that they maintained in the seventeenth century.

The earliest mathematical work to appear in the New World, however, antedated Hodder and Greenwood by more than a century and a half. It was published long before the Puritans had any idea of migrating to another continent, and fifty years before Henry Hudson discovered the river that bears his name. Of this work, known as the *Sumario Compendioso*, there remain perhaps only four copies, and it is desirable, not alone because of its rarity but because of its im-

[1] Address delivered before The Mathematical Association of America, and the section on the History of Science of the American Association for the Advancement of Science, at the University of Chicago, December 28, 1920. The extracts are from a fac-simile reprint of the original work soon to published by Ginn & Company, Boston, with translation and notes by Professor Smith.




¶ Sumario cõpẽdioso delas quẽtas de plata y oro q̃ en los reynos del Piru son necessarias a los mercaderes: y todo genero de tratantes. Cõ algunas reglas tocantes al Arithmetica.

Fecho por Juan Diez freyle.

Title page of the first work on mathematics printed in the New World, Mexico City, 1556.

portance in the history of education on the American continent, that some record of its contents should be made known to scholars.

In order to understand the *Sumario Compendioso* it is necessary to consider briefly the political and social situation in Mexico in the middle of the sixteenth century. Cortés entered the ancient city of Tenochtitlan, later known as Mexico, in the year 1519, but its capture and destruction occurred two years later, in 1521. Thus, in the very year that Luther was attacking certain ancient customs and privileges in the Old World, the representatives of other ancient customs and privileges were attacking and destroying a worthy civilization in the newly discovered continent.

The first viceroy of New Spain, which included the present Mexico, was a man of remarkable genius and of prophetic vision,—Don Antonio de Mendoza. He assumed his office in 1535, and for fifteen years administered the affairs of the colony with such success as to win for himself the name of "the good viceroy." He founded schools, established a mint, ameliorated the condition of the natives, and encouraged the development of the arts. In his efforts at improving the condition of the people he was ably assisted by Juan de Zumárraga, the first Bishop of Mexico. Among the various activities of these leaders was the arrangement made with the printing establishment of Juan Cromberger of Seville whereby a branch should be set up in the capital of New Spain.

As a result of this arrangement there was sent over as Cromberger's representative one Juan Pablos, a Lombard printer, and so the "casa de Juan Cromberger" was established, prepared to spread the doctrines of the Church to the salvation of the souls of the unbelievers. Cromberger himself never went to Mexico, but his name appears either on the *portadas* or in the colophons of all the early books. From and after 1545, however, the name is no longer seen, Cromberger having already died in 1540.

The author of the *Sumario* was one Juan Diez, a native of the Spanish province of Galicia, a companion of Cortés in the conquest of New Spain, and the editor of the works of Juan de Avila, "the apostle of Andalusia," and of the *Itinerario* of the Spanish fleet to Yucatan in 1518. He is sometimes confused with another Juan Diaz (the name being spelled both ways), a contemporary theologian and author. In a letter written to Charles V in 1533 he is mentioned as a "clérigo anciano y honrado," so that he must have been advanced in years when the *Sumario* appeared. That this was the case is also apparent from a record of the expedition of 1518 in which it is stated that "triximus vn clerigo que dezia joan diaz," doubtless a young and adventurous apostle, full of zeal and desire to make known the gospel in the New World.

Juan Diez undertook the work primarily for the purpose of assisting those who were engaged in the buying of the gold and silver which was already being taken from the mines of Peru and Mexico for the further enriching of the moneyed class and the rulers of Spain. He felt that he could best serve this purpose by preparing such a set of tables as should relieve these merchants as far as possible from any necessity for computation. Apparently, however, he was prompted by

the further demand for a brief treatment of arithmetic which should be suited to the needs of apprentices in the counting houses of the New World, and so he devotes eighteen pages to the subject of computation and presents it in a manner not unworthy of the European writers of the period.

The most interesting feature of the work, however, is neither the tables nor the arithmetic; it consists of six pages devoted to algebra, chiefly relating to the quadratic equation.

The book consists of one hundred and three folios, generally numbered. After the dedication (folios i, *v*, and ij, *r*) there is an elaborate set of tables, including those relating to the purchase price of various grades of silver (folio iij, *v*), to per cents (folio xlix, *r*), to the purchase price of gold (folio lvij, *v*), to assays (folio lxxxj, *r*), and to monetary affairs of various kinds. The mathematical text (folio xcj, *v*) consists of twenty-four pages besides the colophon (folio ciij, *v*). As already stated, eighteen of these pages relate chiefly to arithmetic, and six to algebra.

The book was printed in the City of Mexico in the year 1556, being the first work on mathematics to be printed outside the boundaries of Europe, except for the ancient block books of China.

In order to give some idea of the general nature of the work, a few of the problems will be set forth, chiefly those which illustrate the application of algebra as we understand the term today.

**II. Typical Problems not listed under Algebra.** 1. I bought 10 varas of velvet at 20 pesos less than cost, for 34 pesos plus a vara of velvet. How much did it cost a vara?

*Rule:* Add 20 pesos to 34 pesos, making 54 pesos, which will be your dividend. Subtract one from 10 varas, leaving 9. Divide 54 by 9, giving 6, the price per vara.

*Proof:* 10 varas at 6 pesos is 60 pesos. This minus 20 pesos is 40. You paid 34 pesos plus a vara costing 6 pesos, and this gives the result, 40 pesos.

2. I bought 9 varas of velvet for as much more than 40 pesos as 13 varas at the same price is less than 70 pesos. How much did a vara cost?

*Rule:* Add the pesos, 40 and 70, making 110. Add the varas, 9 and 13, making 22. Dividing 110 by 22 the quotient is 5, the price of each vara.

*Proof:* 9 varas at 5 pesos is 45 pesos, which is 5 more than 40 pesos; and 13 varas at 5 pesos is 65, which is 5 pesos less than 70, as you see.

3. Required a number which if 8 is added to it will be a square, and if 8 is subtracted from it will also be a square. Take half of eight, which is 4; square it, making 16; add 1, making 17, and this is the number to which if you add 8 you have 25, the root of which is 5; and if 8 is taken from it there is left 9, the root of which is 3; for 3 times 3 is 9, as you see.

4. Find 2 numbers the sum of the squares of which will make a square number which has an integral root. The first numbers are 3 and 4, for their squares are 9 and 16, and these added together make 25, the root of which is 5. Observe that

you have 5 numbers; the first are 2 and 3; the next are 3 and 4, the proposed numbers; and there is also 5, which is their root. Place these numbers as you see in the figure below. Then use cross multiplication, saying "3 times 3 is 9, and 2 times 4 is 8." Place these numbers at the right-hand side, one under the other. Then multiply again at the top, 2 times 3 is 6; and underneath, 3 times 4 is 12. Now subtract the less from the greater, that is, 6 from 12, and there remains 6. Divide this by 5, the root of the assumed numbers, and the quotient is $1\frac{1}{5}$, one of the numbers required. Now add 8 and 9, the products of the first multiplication, and the sum is 17. Divide this by 5 and the quotient is $3\frac{2}{5}$, and this is the second required number.

*Proof:* The square of $1\frac{1}{5}$ is $1\frac{11}{25}$; the square of $3\frac{2}{5}$ is $11\frac{14}{25}$; and these added together, as you see, make 13.

$$5\ \begin{array}{cccc} & 6 & & \\ 2 & 6 & 3 & 9 \\ \hline 3 & & 4 & 8 \\ & 12 & & \end{array}\ 17 \qquad\qquad \begin{array}{l} 02 \\ 17 \mid 3\frac{2}{5} \\ 5 \end{array} \qquad \begin{array}{l} 1 \\ 6 \mid 1\frac{1}{5} \\ 5 \end{array}$$

**III. Typical Problems listed under Algebra.** Although the above problems are solved by arithmetical rules, they are essentially algebraic. Under the title *Arte Mayor* the author gives a number of examples generally involving quadratic equations, of which the following are types:

1. Find a square from which if $15\frac{3}{4}$ is subtracted the result is its own root.

*Rule:* Let the number be *cosa* ($x$). The square of half a *cosa* is equal to $\frac{1}{4}$ of a *zenso* ($x^2$). Adding 15 and $\frac{3}{4}$ to $\frac{1}{4}$ makes 16, of which the root is 4, and this plus $\frac{1}{2}$ is the root of the required number.

*Proof:* Square the square root of 16, plus half a *cosa*, which is four and a half, giving 20 and $\frac{1}{4}$, which is the square number required. From $20\frac{1}{4}$ subtract 15 and $\frac{3}{4}$ and you have 4 and $\frac{1}{2}$, which is the root of the number itself.

2. A man takes passage in a ship and asks the master what he has to pay. The master says that it will not be any more than for the others. The passenger on again asking how much it would be, the master replies: "It will be the number of pesos which, multiplied by itself and added to the number, will give 1260." Required to know how much the master asked.

*Rule:* Let the cost be a *cosa* of pesos. Then half of a *cosa* squared makes $\frac{1}{4}$ of a *zenso*, and this added to 1260 makes 1260 and a quarter, the root of which less $\frac{1}{2}$ of a *cosa* is the number required. Reduce 1260 and $\frac{1}{4}$ to fourths; this is equal to $\frac{5041}{4}$, the root of which is 71 halves; subtract from it half of a *cosa* and there remains 70 halves, which is equal to 35 pesos, and this is what was asked for the passage.

*Proof:* Multiply 35 by itself and you have 1225; adding to it 35, you have 1260, the required number.

3. A man is selling goats. The number is unknown except that it is stated that a merchant asked how many there were and the seller replied: "There are

so many that, the number being squared and the product quadrupled, the result will be 90,000." Required to know how many goats he had.[1]

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# A DETERMINATION OF THE CURVE MINIMIZING THE AREA ENCLOSED BY IT AND ITS EVOLUTE.

By OTTO DUNKEL, Washington University, St. Louis, Mo.

One of the problems treated in the calculus of variations as an example in which the second derivative appears in the integrand is that of determining the curve in a plane passing through two fixed points which with its evolute and its two normals of given directions at these two points enclose a minimum area.[2] The special form of the integrand permits this problem to be solved without resorting to the general theory of the calculus of variations, and the conditions appear as necessary and sufficient simultaneously. The solution given here appears to be new so far as the writer can learn from the references consulted. It is adapted only to this special form of the integrand, but it is quite possible that there may be other problems of this form to which it applies. For example, the problem of determining the curve which with its caustic produced by parallel rays of light, and the reflected rays at two of its given points, enclose a minimum area may also be solved in the same way.[3] It will be observed that the method admits of a slight generalization.

**The Necessary and Sufficient Conditions for a Minimum.** Let one of the fixed points be the origin and the other $(x_2, y_2)$ and let the inclination of the curve to the $y$-axis at these two points be $\theta_1$ and $\theta_2$, respectively, $\theta_2 > \theta_1$. Let $s$ be the length of the arc of the curve measured from the origin and, $R$, the radius of curvature. Then

$$R = \frac{ds}{d\theta} \tag{1}$$

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[1] In the above brief extracts the archaic forms of expression have been retained so far as the circumstances of translation permit. No effort has been made to explain, in this presentation, the method of attacking the problems, or to consider the sources from which the author drew his materials.

[2] This problem is discussed in many well-known works, for example: I. Todhunter, *Researches in the Calculus of Variations*, London, 1871, chapter 13; H. Hancock, *Lectures on the Calculus of Variations*, Cincinnati, 1904, pp. 75–76; A. Kneser, *Lehrbuch der Variationsrechnung*, Braunschweig, 1900, pp. 203, 219; and O. Bolza, *Vorlesungen über Variationsrechnung*, Leipzig, 1909, p. 152. It originated with Euler (*Methodus Inveniendi Lineas Curvas* . . . Lausannæ & Genevæ, 1744, pp. 64–66): "Invenire curvam *Am*, quæ cum sua evoluta *AR* & radio osculi *mR* in quavis loco applicato, minimum spatium *ARm* includat"; the solution which Euler gives employs the calculus of variations. In *Annals of Mathematics*, new series, vol. 14, pp. 14–26, 1912, E. J. Miles discusses the "Determination of the constants in Euler's problem concerning the minimum area between a curve and its evolute," and in this MONTHLY, *1917*, 420–422, P. R. Rider discusses "An intrinsic equation solution of a problem of Euler."

[3] A paper on this problem was read by Professor Dunkel before the American Mathematical Society, November 27, 1920.—EDITOR..